Model Solutions

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Thursday 14 May 2020

Afternoon (Time: 1 hour 30 minutes)	Paper Reference **9FM0/02**

Further Mathematics

Advanced

Paper 2: Core Pure Mathematics 2

You must have: Mathematical Formulae and Statistical Tables (Green), calculator	Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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Pearson

1. The curve C has equation

$$y = 31\sinh x - 2\sinh 2x \qquad x \in \mathbb{R}$$

Determine, in terms of natural logarithms, the exact x coordinates of the stationary points of C.

(7)

$y = 31\sinh x - 2\sinh 2x$

Differentiate y:

$\frac{dy}{dx} = 31\cosh x - 4\cosh 2x$

As $\cosh 2x = 2\cosh^2 x - 1$

$\frac{dy}{dx} = 31\cosh x - 4(2\cosh^2 x - 1)$

$= 31\cosh x - 8\cosh^2 x + 4$

Stationary point so let $\frac{dy}{dx} = 0$

$0 = 8\cosh^2 x - 31\cosh x - 4$

$0 = (8\cosh x + 1)(\cosh x - 4)$

$\cosh x = -\frac{1}{8}$ (reject as $\cosh x \geq 1$)

$\cosh x = 4$

As $\cosh x = \frac{e^x + e^{-x}}{2}$

So $\frac{e^x + e^{-x}}{2} = 4$

$e^x + e^{-x} = 8$

Multiply both sides by e^x

$e^{2x} + 1 = 8e^x$

$e^{2x} - 8e^x + 1 = 0$

$e^x = 4 \pm \sqrt{15}$

$\boxed{x = \ln(4 \pm \sqrt{15})}$

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Question 1 continued

(Total for Question 1 is 7 marks)

2. In an Argand diagram, the points A and B are represented by the complex numbers $-3 + 2\text{i}$ and $5 - 4\text{i}$ respectively. The points A and B are the end points of a diameter of a circle C.

(a) Find the equation of C, giving your answer in the form

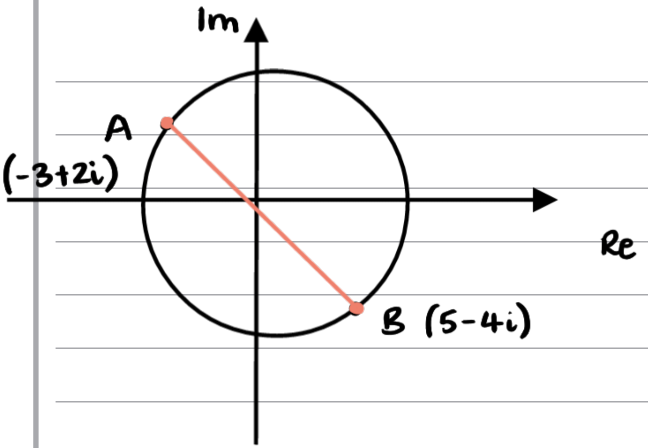
$$|z - a| = b \qquad a \in \mathbb{C},\ b \in \mathbb{R}$$

(3)

The circle D, with equation $|z - 2 - 3\text{i}| = 2$, intersects C at the points representing the complex numbers z_1 and z_2

(b) Find the complex numbers z_1 and z_2

(6)



Diameter = distance between A and B

$$\text{Diameter} = \sqrt{(-3-5)^2 + (2 - -4)^2}$$
$$= \sqrt{(-8)^2 + (6)^2}$$
$$= 10$$

$$\text{Radius} = \frac{10}{2} = 5$$

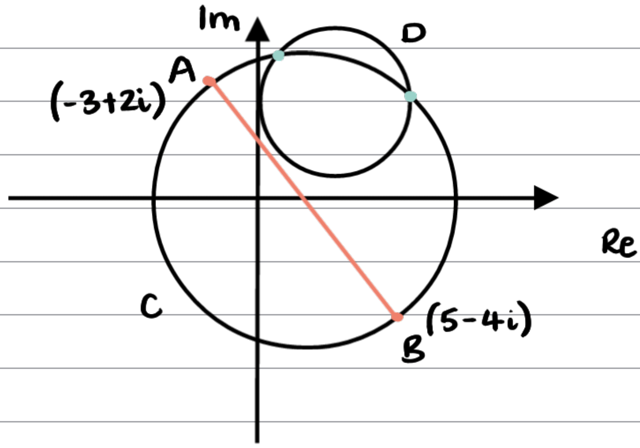
$$\text{centre} = \left(\frac{5+-3}{2}, \frac{-4+2}{2}\right)$$
$$= (1, -1)$$

In the form $|z - a| = b$
where $a = 1 - i$ and $b = 5$

$$|z - (1 - i)| = 5$$
$$|z - 1 + i| = 5$$

Question 2 continued

b)



Rewrite $|z-1+i|=5$ as a cartesian equation:

$(x-1)^2+(y+1)^2=5^2$

$(x-1)^2+(y+1)^2=25$

$x^2-2x+1+y^2+2y+1=25$

$x^2-2x+y^2+2y=23 \quad -(1)$

Rewrite $|z-2-3i|=2$ as a cartesian equation:

$(x-2)^2+(y-3)^2=2^2$

$(x-2)^2+(y-3)^2=4$

$x^2-4x+4+y^2-6y+9=4$

$x^2-4x+y^2-6y=-9 \quad -(2)$

(1) − (2)

$(x^2-2x+y^2+2y)-(x^2-4x+y^2-6y)=23-(-9)$

$2x+8y=32$

$2x=-8y+32$

$x=-4y+16$

Sub $x=-4y+16$ into circle D equation:

$((-4y+16)-2)^2+(y-3)^2=4$

$(-4y+14)^2+(y-3)^2=4$

$16y^2-112y+196+y^2-6y+9=4$

$17y^2-118y+201=0$

$y=\frac{67}{17} \qquad y=3$

Question 2 continued

Sub values of y into $x = -4y + 16$:

when $y = 67/17$

$x = -4\left(\frac{67}{17}\right) + 16$

$= \frac{4}{17}$

when $y = 3$

$x = -4(3) + 16$

$= 4$

Complex numbers

$4 + 3i$, $\frac{4}{17} + \frac{67}{17}i$

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Question 2 continued

(Total for Question 2 is 9 marks)

3. A scientist is investigating the concentration of antibodies in the bloodstream of a patient following a vaccination.
The concentration of antibodies, x, measured in micrograms (μg) per millilitre (ml) of blood, is modelled by the differential equation

$$100\frac{d^2x}{dt^2} + 60\frac{dx}{dt} + 13x = 26$$

where t is the number of weeks since the vaccination was given.

(a) Find a general solution of the differential equation.

(4)

Initially,
- there are no antibodies in the bloodstream of the patient
- the concentration of antibodies is estimated to be increasing at 10 μg/ml per week

(b) Find, according to the model, the maximum concentration of antibodies in the bloodstream of the patient after the vaccination.

(8)

A second dose of the vaccine has to be given to try to ensure that it is fully effective. It is only safe to give the second dose if the concentration of antibodies in the bloodstream of the patient is less than 5 μg/ml.

(c) Determine whether, according to the model, it is safe to give the second dose of the vaccine to the patient exactly 10 weeks after the first dose.

(2)

a) $100m^2 + 60m + 13 = 0$

$m = -0.3 \pm 0.2i$

$\therefore\ x = e^{-0.3t}(A\cos 0.2t + B\sin 0.2t)$

To work out PI :

let $x = \lambda$

$\frac{dx}{dt} = 0$

$\frac{d^2x}{dt^2} = 0$

sub values into $100\frac{d^2x}{dt^2} + 60\frac{dx}{dt} + 13x = 26$

$0 + 60(0) + 13(\lambda) = 26$

$13\lambda = 26$

$\lambda = 2$

$\therefore$ PI : $x = 2$

General Solution : $x = e^{-0.3t}(A\cos 0.2t + B\sin 0.2t) + 2$

Question 3 continued

b) when $t=0$, $x=0$

$0 = A + 2$

$\therefore A = -2$

Differentiate x:

$$\frac{dx}{dt} = -0.3e^{-0.3t}(A\cos 0.2t + B\sin 0.2t) + e^{-0.3t}(0.2B\cos 0.2t - 0.2A\sin 0.2t)$$

Subbing $A = -2$ into $\frac{dx}{dt}$

$$\frac{dx}{dt} = -0.3e^{-0.3t}(B\sin 0.2t - 2\cos 0.2t) + e^{-0.3t}(0.2B\cos 0.2t + 0.4\sin 0.2t)$$

when $\frac{dx}{dt} = 10$, $t = 0$

$10 = -0.3(-2) + 0.2B$

$9.4 = 0.2B$

$B = 47$

Subbing values of A and B into the general equation:

$$x = e^{-0.3t}(47\sin 0.2t - 2\cos 0.2t) + 2$$

To find the maximum concentration of antibodies, let $\frac{dx}{dt} = 0$:

$$0 = -0.3e^{-0.3t}(47\sin 0.2t - 2\cos 0.2t) + e^{-0.3t}(9.4\cos 0.2t + 0.4\sin 0.2t)$$

$$0.3\cancel{e^{-0.3t}}(47\sin 0.2t - 2\cos 0.2t) = \cancel{e^{-0.3t}}(9.4\cos 0.2t + 0.4\sin 0.2t)$$

$14.1\sin 0.2t - 0.6\cos 0.2t = 9.4\cos 0.2t + 0.4\sin 0.2t$

$13.7\sin 0.2t = 10\cos 0.2t$

$\tan 0.2t = \frac{10}{13.7}$

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Question 3 continued

$0.2t = 0.6305301382$

$t = 3.152650691$ weeks

Sub $t = 3.15...$ into x:

$x = e^{-0.3(3.15...)}\left(47 \times \sin(0.2 \times (3.15...)) - 2\cos(0.2 \times (3.15...))\right) + 2$

$= 12.1$ μg/ml (3sf)

c) when $t = 10$

$x = e^{-0.3(10)}\left(47\sin(0.2 \times 10) - 2\cos(0.2 \times 10)\right) + 2$

$= 4.16...$

$4.16 < 5$

∴ The model suggests that it would be safe to give the second dose.

Question 3 continued

(Total for Question 3 is 14 marks)

4. (a) Use de Moivre's theorem to prove that

$$\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$$

(5)

(b) Hence find the distinct roots of the equation

$$1 + 7x - 56x^3 + 112x^5 - 64x^7 = 0$$

(5)

giving your answer to 3 decimal places where appropriate.

a) $(\cos\theta + i\sin\theta)^7 = \cos^7\theta + 7i\cos^6\theta\sin\theta - 21\cos^5\theta\sin^2\theta$
$- 35i\cos^4\theta\sin^3\theta + 35\cos^3\theta\sin^4\theta$
$+ 21i\cos^2\theta\sin^5\theta - 7\cos\theta\sin^6\theta - i\sin^7\theta$

$i\sin 7\theta = \mathrm{Im}((\cos\theta + i\sin\theta)^7)$
$= 7i\cos^6\theta\sin\theta - 35i\cos^4\theta\sin^3\theta + 21i\cos^2\theta\sin^5\theta$
$- i\sin^7\theta$

$\sin 7\theta = 7\cos^6\theta\sin\theta - 35\cos^4\theta\sin^3\theta + 21\cos^2\theta\sin^5\theta$
$- \sin^7\theta$
$= 7(1-\sin^2\theta)^3\sin\theta - 35(1-\sin^2\theta)^2\sin^3\theta$
$+ 21(1-\sin^2\theta)\sin^5\theta - \sin^7\theta$
$= 7\sin\theta(1 - 3\sin^2\theta + 3\sin^4\theta - \sin^6\theta)$
$- 35\sin^3\theta(1 - 2\sin^2\theta + \sin^4\theta)$
$+ 21\sin^5\theta(1 - \sin^2\theta)$
$- \sin^7\theta$
$= 7\sin\theta - 21\sin^3\theta + 21\sin^5\theta - 7\sin^7\theta$
$- 35\sin^3\theta + 70\sin^5\theta - 35\sin^7\theta$
$+ 21\sin^5\theta - 21\sin^7\theta - \sin^7\theta$
$= 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$

$\sin 7\theta = 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$

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Question 4 continued

b) Let $\sin\theta = x$

$\therefore 7\sin\theta - 56\sin^3\theta + 112\sin^5\theta - 64\sin^7\theta$

$= 7x - 56x^3 + 112x^5 - 64x^7 \quad = \sin 7\theta$

$\Rightarrow \sin 7\theta + 1 = 1 + 7x - 56x^3 + 112x^5 - 64x^7$

$\sin 7\theta + 1 = 0$

$\sin 7\theta = -1$

$7\theta = -\frac{5\pi}{2}, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \ldots$

$\theta = -\frac{5\pi}{14}, -\frac{\pi}{14}, \frac{3\pi}{14}, \frac{\pi}{2}, \ldots$

as $\sin\theta = x$

$x = -0.901, -0.223, 0.623, 1$ (3dp)

Question 4 continued

Question 4 continued

(Total for Question 4 is 10 marks)

5. (a) $$y = \tan^{-1} x$$

Assuming the derivative of $\tan x$, prove that

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$

(3)

$$f(x) = x\tan^{-1} 4x$$

(b) Show that

$$\int f(x)\,dx = Ax^2 \tan^{-1} 4x + Bx + C\tan^{-1} 4x + k$$

where k is an arbitrary constant and A, B and C are constants to be determined.

(5)

(c) Hence find, in exact form, the mean value of f(x) over the interval $\left[0, \frac{\sqrt{3}}{4}\right]$

(2)

a) as $y = \tan^{-1} x$

$\therefore \; x = \tan y$

differentiate $x = \tan y$

$$\frac{dx}{dy} = \sec^2 y$$

As $\sec^2 y = 1 + \tan^2 y$:

$$\frac{dx}{dy} = 1 + \tan^2 y$$

$$\frac{dy}{dx} = \frac{1}{1+\tan^2 y}$$

As $x = \tan y$

$$\frac{dy}{dx} = \frac{1}{1+x^2}$$ (as required)

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Question 5 continued

b) $f(x) = x\tan^{-1}4x$

Using part a :

$$\frac{d}{dx}\left(\tan^{-1}4x\right) = 4 \times \frac{1}{1+(4x)^2}$$

$$= \frac{4}{1+16x^2}$$

$$\int x\tan^{-1}4x \, dx$$

Using integration by parts:

$uv - \int u'v$

let $u = \tan^{-1}(4x)$ let $v' = x$

$u' = \dfrac{4}{1+16x^3}$ $v = \dfrac{x^2}{2}$

$$\Rightarrow \frac{x^2}{2}\tan^{-1}(4x) - \int \frac{4}{1+16x^2} \times \frac{x^2}{2} \, dx$$

$$= \frac{x^2}{2}\tan^{-1}(4x) - \underbrace{\int 2x^2(1+16x^2)^{-1} \, dx}_{\text{Equation 1}}$$

Question 5 continued

Equation 1 :

$$\int \frac{2x^2}{1+16x^2}\,dx = \frac{1}{8}\int \frac{16x^2}{1+16x^2}\,dx$$

$$= \frac{1}{8}\int \frac{(16x^2+1)-1}{16x^2+1}\,dx$$

$$= \frac{1}{8}\int 1 - \frac{1}{16x^2+1}\,dx$$

As $\int \frac{1}{16x^2-1}\,dx = \frac{1}{4}\tan^{-1}4x + c$

$$\Rightarrow \frac{1}{8}\left[x - \frac{1}{4}\tan^{-1}4x\right] + c$$

$$= \frac{1}{8}x - \frac{1}{32}\tan^{-1}4x + c$$

$$\therefore \int x\tan^{-1}4x\,dx =$$

$$\frac{x^2}{2}\tan^{-1}4x - \left(\frac{1}{8}x - \frac{1}{32}\tan^{-1}4x\right) + K$$

$$= \frac{x}{2}\tan^{-1}4x - \frac{1}{8}x + \frac{1}{32}\tan^{-1}4x + K$$

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Question 5 continued

c) mean value $= \left(\dfrac{1}{\frac{\sqrt{3}}{4} - 0}\right) \displaystyle\int_0^{\frac{\sqrt{3}}{4}} x\tan^{-1} 4x \, dx$

$$= \frac{4}{\sqrt{3}} \left[\frac{x}{2}\tan^{-1}4x - \frac{1}{8}x + \frac{1}{32}\tan^{-1} 4x \right]_0^{\frac{\sqrt{3}}{4}}$$

$$= \frac{4}{\sqrt{3}} \left[\left(\left(\frac{3}{32} \times \frac{\pi}{3} \right) - \left(\frac{1}{8} \times \frac{\sqrt{3}}{4} \right) + \left(\frac{1}{32} \times \frac{\pi}{3} \right) \right) - 0 \right]$$

$$= \frac{\sqrt{3}}{72}\left(4\pi - 3\sqrt{3}\right)$$

(Total for Question 5 is 10 marks)

6.

$$\mathbf{M} = \begin{pmatrix} k & 5 & 7 \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Given that $k \neq 4$, find, in terms of k, the inverse of the matrix $\mathbf{M}$.

(4)

(b) Find, in terms of p, the coordinates of the point where the following planes intersect.

$$\begin{aligned} 2x + 5y + 7z &= 1 \\ x + y + z &= p \\ 2x + y - z &= 2 \end{aligned}$$

(3)

(c) (i) Find the value of q for which the following planes intersect in a straight line.

$$\begin{aligned} 4x + 5y + 7z &= 1 \\ x + y + z &= q \\ 2x + y - z &= 2 \end{aligned}$$

(ii) For this value of q, determine a vector equation for the line of intersection.

(7)

a) $\det M = K((1\times -1)-(1\times 1)) - 5((1\times -1)-(1\times 2)) + 7((1\times 1)-(2\times 1))$

$= -2K + 15 - 7$

$= -2K + 8 \quad = 8 - 2K$

Minor: $\begin{pmatrix} -2 & -3 & -1 \\ -12 & -K-14 & K-10 \\ -2 & K-7 & K-5 \end{pmatrix}$

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Question 6 continued

As $\begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

Cofactors:

$$\begin{pmatrix} -2 & 3 & -1 \\ 12 & -k-14 & 10-k \\ -2 & 7-k & k-5 \end{pmatrix}$$

$$M^{-1} = \frac{1}{\det M} \begin{pmatrix} -2 & 12 & -2 \\ 3 & -k-14 & 7-k \\ -1 & 10-k & k-5 \end{pmatrix}$$

$$= \frac{1}{8-2k} \begin{pmatrix} -2 & 12 & -2 \\ +3 & -k-14 & 7-k \\ -1 & 10-k & k-5 \end{pmatrix}$$

b) let $k=2$

$$M^{-1} = \frac{1}{4} \begin{pmatrix} -2 & 12 & -2 \\ 3 & -16 & 5 \\ -1 & 8 & -3 \end{pmatrix}$$

Question 6 continued

$$\begin{pmatrix} 2 & 5 & 7 \\ 1 & 1 & 1 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ p \\ 2 \end{pmatrix}$$

As $M^{-1}M\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M^{-1}\begin{pmatrix} 1 \\ q \\ 2 \end{pmatrix}$:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = M^{-1}\begin{pmatrix} 1 \\ p \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4}\begin{pmatrix} -2 & 12 & -2 \\ 3 & -16 & 5 \\ -1 & 8 & -3 \end{pmatrix}\begin{pmatrix} 1 \\ p \\ 2 \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} (-2\times 1) + (12\times p) + (-2\times 2) \\ (3\times 1) + (-16\times p) + (5\times 2) \\ (-1\times 1) + (8\times p) + (-3\times 2) \end{pmatrix}$$

$$= \frac{1}{4}\begin{pmatrix} 12p - 6 \\ -16p + 13 \\ 8p - 7 \end{pmatrix}$$

$$= \begin{pmatrix} 3p - \frac{3}{2} \\ \frac{13}{4} - 4p \\ 2p - \frac{7}{4} \end{pmatrix} = \left(3p - \frac{3}{2}, \frac{13}{7} - 4p, 2p - \frac{7}{4}\right)$$

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Question 6 continued

c i) let $k = 4$

$4x + 5y + 7z = 1$ — ①
$x + y + z = q$ — ②
$2x + y - z = 2$ — ③

Eliminate z :

② + ③

$(x+y+z) + (2x+y-z) = q+2$

$3x + 2y = q+2$ — ④

① + 7 × ③

$(4x+5y+7z) + (14x+7y-7z) = 1 + 14$

$18x + 12y = 15$ — ⑤

Eliminate x/y :

⑤ − 6 × ④

$(18x+12y) - (18x+12y) = 15 - 6q - 12$

$0 = 3 - 6q$

$6q = 3$

$q = \frac{1}{2}$

(Total for Question 6 is 14 marks)

c ii) Using c i) :

Let $x = \lambda$ and $q = \frac{1}{2}$

Sub $x = \lambda$ in 4

$$3\lambda + 2y = \frac{5}{2}$$

$$2y = \frac{5}{2} - 3\lambda$$

$$y = \frac{5 - 6\lambda}{4}$$

Eliminating y :

3 − 2

$$(2x + y - z) - (x + y + z) = 2 - q$$

$$x - 2z = \frac{3}{2} \quad - 6$$

Sub $x = \lambda$ into 6

$$\lambda - 2z = \frac{3}{2}$$

$$2z = \lambda - \frac{3}{2}$$

$$z = \frac{2\lambda - 3}{4}$$

∴ Vector equation : $\boxed{\begin{pmatrix} 0 \\ 5/4 \\ -3/4 \end{pmatrix} + t\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}}$

7.

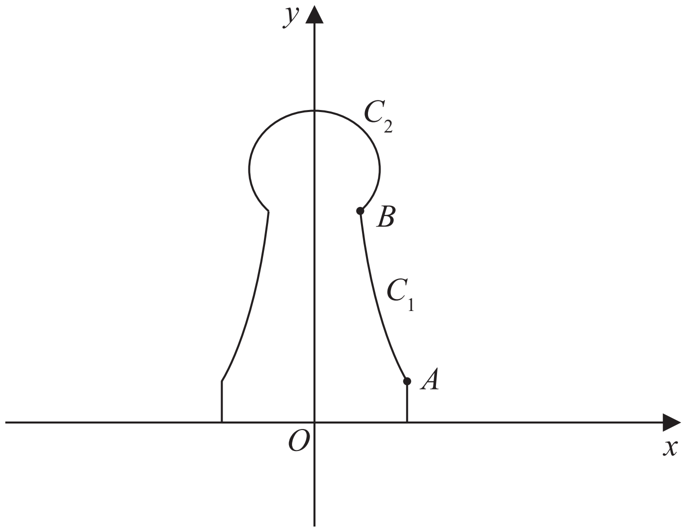

Figure 1

A student wants to make plastic chess pieces using a 3D printer. Figure 1 shows the central vertical cross-section of the student's design for one chess piece. The plastic chess piece is formed by rotating the region bounded by the y-axis, the x-axis, the line with equation $x = 1$, the curve C_1 and the curve C_2 through 360° about the y-axis.

The point A has coordinates (1, 0.5) and the point B has coordinates (0.5, 2.5) where the units are centimetres.

The curve C_1 is modelled by the equation

$$x = \frac{a}{y+b} \qquad 0.5 \leqslant y \leqslant 2.5$$

(a) Determine the value of a and the value of b according to the model.

(2)

The curve C_2 is modelled to be an arc of the circle with centre (0, 3).

(b) Use calculus to determine the volume of plastic required to make the chess piece according to the model.

(9)

a) when $x=1$, $y=0.5$

$\therefore \; 1 = \dfrac{a}{0.5+b} \Rightarrow a = 0.5+b$

when $x = 0.5$, $y = 2.5$

$0.5 = \dfrac{a}{2.5+b} \Rightarrow 1.25 + 0.5b = a$

Question 7 continued

$\therefore\ 1.25 + 0.5b = 0.5 + b$

$0.75 = 0.5b$

$b = 1.5$

$\therefore\ a = 0.5 + 1.5$

$a = 2$

$a = 2,\ b = 1.5$ $\qquad \therefore\ x = \dfrac{2}{y + 1.5}$

b)

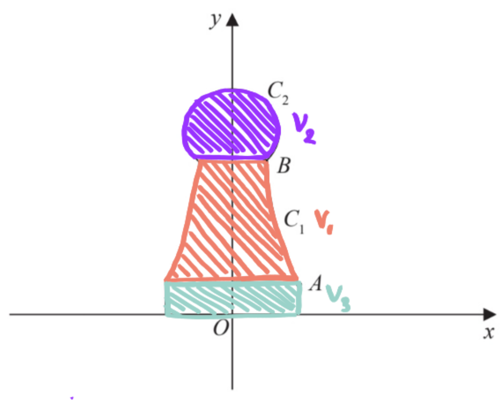

Figure 1

Working out V_3 (cylinder)

$V_1 = \pi r^2 h$

$= \pi \times 1^2 \times 0.5$

$= \dfrac{\pi}{2}$

Working out V_1

$\pi \int_{0.5}^{2.5} x^2 \, dy = \pi \int_{0.5}^{2.5} \left(\dfrac{2}{y+1.5}\right)^2 dy$

$= \pi \int_{0.5}^{2.5} \dfrac{4}{(y+1.5)^2} \, dy$

$= 4\pi \left[-(y+1.5)^{-1} \right]_{0.5}^{2.5}$

P 6 2 6 7 1 A 0 2 5 2 8

Question 7 continued

$$= 4\pi\left[-\left(\frac{1}{4}\right) - -\left(\frac{1}{2}\right)\right]$$

$$= \pi$$

Working out V_2

Work out the circle's equation

- centre $(0, 3)$
- Radius $\Rightarrow \sqrt{(0.5-0)^2 + (2.5-3)^2} = \frac{\sqrt{2}}{2}$

$\therefore x^2 + (y-3)^2 = \frac{1}{2}$

Finding the limits for the integral.

when $x = 0$

$(y-3)^2 = \frac{1}{2}$

$y - 3 = \frac{\sqrt{2}}{2}$

$y = 3 + \frac{\sqrt{2}}{2}$

$$\pi\int_{2.5}^{3+\frac{\sqrt{2}}{2}} x^2 \, dy = \pi\int_{2.5}^{3+\frac{\sqrt{2}}{2}} \left(\frac{1}{2} - (y-3)^2\right) dy$$

$$= \pi\left[\frac{y}{2} - \frac{1}{3}(y-3)^3\right]_{2.5}^{3+\frac{\sqrt{2}}{2}}$$

$$= \pi\left[\left(\frac{6+\sqrt{2}}{4} - \frac{1}{3}\left(\frac{\sqrt{2}}{2}\right)^3\right) - \left(\frac{2.5}{2} - \frac{1}{3}(-0.5)^3\right)\right]$$

Question 7 continued

$$= \left(\frac{5+4\sqrt{2}}{24}\right)\pi$$

Adding V_1, V_2 and V_3 together to get the total Volume:

$$\pi + \left(\frac{5+4\sqrt{2}}{24}\right)\pi + \frac{\pi}{2} \approx 6.11 \text{ cm}^3 \text{ (3sf)}$$

Question 7 continued

(Total for Question 7 is 11 marks)

TOTAL FOR PAPER IS 75 MARKS